Our final measurements showed typical insertion loss and isolation characteristics as given in Fig. 2. At 258.2 GHz[1] we measured for circulator no. 1:

Port	500 G	Bias Field 500 G (opposite circulation)	0 G
1—2	1.8 dB	21 dB	8.2 dB
1—3	27 dB	2.0 dB	8.0 dB
2—3	2.1 dB	27 dB	8.0 dB
2—1	21 dB	2.3 dB	7.4 dB
3—1	1.5 dB	17 dB	6.6 dB
3—2	24 dB	2.5 dB	8.0 dB

The lack of good loads[3] and VSWR measuring equipment prevented high accuracy (± 0.2 dB) of the measurements. The mm wave power was generated by a second harmonic generator (point-contact silicon diode) driven by a VARIAN VC714 klystron.

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Beam Waveguide Excitation by the Aperture Field of a Tubular Waveguide

Abstract—**The excitation of a lens-type beam waveguide by the aperture field of a conventional waveguide of circular cross section is treated, assuming that a superposition of an H_{11}-mode and a E_{11}-mode is propagating in the metallic waveguide. The launching efficiency for the dominant beam mode depends on the amplitude ratio of the H_{11}- and E_{11}-modes and on the ratio of the beam mode parameter to the radius of the tubular waveguide. If both quantities are chosen appropriately a theoretical launching efficiency of 98.3 percent can be achieved.**

Discussion

The excitation of a lens-type beam waveguide by a conventional metallic waveguide was first studied by Baskakow.[1] He considers the structure shown in Fig. 1. A metallic waveguide of circular cross section terminates in the plane $z=0$; outside the waveguide aperture this plane is assumed to be covered by a perfectly conducting screen. The plane $z=0$, simultaneously, is the input plane of a beam waveguide with circular lenses. The distance z_0 of the first lens from the plane $z=0$ is half the spacing of the lenses.

Baskakow assumes that, in the metallic waveguide, a field of the H_{11}-type is propagating in the positive z-direction, and calculates the launching efficiency for the dominant beam mode of the beam waveguide. If the lens diameter is sufficiently large, the field of this mode in the plane $z=0$ is linearly polarized, has a Gaussian amplitude distribution, and a plane phase front. The launching efficiency which is defined by the power ratio of the dominant beam mode and the H_{11}-mode depends on the quantity R_0/ρ_0 where R_0 is the radius of the metallic waveguide, and ρ_0 is the mode parameter of the beam waveguide, i.e., the radius at which the energy density in the dominant beam modes has decreased by a factor e. For $R_0/\rho_0=1.84$ the launching efficiency reaches an optimum value of 86.7 percent.[2]

If in the tubular waveguide an E_{11}-mode of appropriate phase and amplitude is superimposed to the H_{11}-mode, the cross-sectional field distribution in the aperture of the waveguide will closely approximate the Gaussian field distribution of the dominant beam mode in the plane $z=0$. This has been demonstrated by Chaffin and Beyer[3] who used a dual mode $(H_{11}+E_{11})$ horn with a phase correcting lens at the aperture.

In the following it will be shown that such dual mode excitation leads to theoretical launching effieiencies for the dominant beam mode of up to 98.3 percent.

If an H_{11}-mode of amplitude A and an E_{11}-mode of amplitude B is propagating in the tubular guide whose radius is so large that the phase velocity of the modes approaches the free space velocity, the distribution of the tangential field components in the plane $z=0$ is

$$E_\rho^{(1)} \approx +\left\{A\bar{\gamma}\frac{J_1(\bar{\gamma}\rho)}{\bar{\gamma}\rho} + B\gamma J_1'(\gamma\rho)\right\} \cdot \cos\phi \approx \sqrt{\frac{\mu}{\epsilon}}\, H_\phi^{(1)}$$

$$0 \le \rho \le R \quad (1)$$

$$E_\phi^{(1)} \approx -\left\{A\bar{\gamma}J_1'(\bar{\gamma}\rho) + B\gamma\frac{J_1(\gamma\rho)}{\gamma\rho}\right\} \cdot \sin\phi \approx -\sqrt{\frac{\mu}{\epsilon}}\, H_\rho^{(1)}$$

where ρ and ϕ are polar coordinates, and $\gamma R_0 = a_{11}$, $\bar{\gamma}R_0 = \bar{a}_{11}$ are the first zeros of the Bessel function $J_{11}(x)$ and its derivative $J_1'(x)$, respectively:

$$a_{11} = 3.8317 \qquad \bar{a}_{11} = 1.8412. \quad (2)$$

On the metallic screen outside the waveguide aperture E_ρ and E_ϕ are zero.

$$E_\rho^{(1)} = E_\phi^{(1)} = 0 \quad R_0 \le \rho < \infty,\ z = 0. \quad (3)$$

If the lenses are sufficiently large, i. e., if

$$\sqrt{\frac{k}{2z_0}}\, R > 2.5$$

the dominant beam mode in the plane $z=0$ has a Gaussian field distribution[4]

$$E_\rho^{(2)} = +e^{-1/2(\rho/\rho_0)^2}\cos\phi = \sqrt{\frac{\mu}{\epsilon}}\, H_\phi^{(2)}$$

$$E_\phi^{(2)} = -e^{-1/2(\rho/\rho_0)^2}\sin\phi = -\sqrt{\frac{\mu}{\epsilon}}\, H_\rho^{(2)}$$

$$0 \le \rho < \infty \quad (4)$$

where ρ_0 is the mode parameter which is determined by the focal length f and the spacing $2z_0$ of the lenses

$$\rho_0^2 = 2\frac{z_0}{k}\sqrt{2\frac{f}{z_0} - 1}. \quad (5)$$

Since the field distribution (4) of the dominant beam mode is a real function, maximum launching efficiency is obtained, if the H_{11}-mode and the E_{11}-mode have the same phase in the plane $z=0$, in other words if the amplitude factors A and B are both real. As the beam modes of the beam waveguide are mutually orthogonal, the amplitude C of the dominant beam mode is given by

$$C = \frac{\int_0^{2\pi}\int_0^{R_0}\left(E_\rho^{(1)}E_\rho^{(2)} + E_\phi^{(1)}E_\phi^{(2)}\right)\rho\, d\rho\, d\phi}{\int_0^{2\pi}\int_0^{\infty}\left(E_\rho^{(2)2} + E_\phi^{(2)2}\right)\rho\, d\rho\, d\phi}. \quad (6)$$

With (1) and (4) this expression can be rewritten as follows

$$C = \frac{A}{\rho_0}\cdot F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right) + \frac{B}{\rho_0}\cdot F\left(a_{11}, \frac{R_0}{\rho_0}\right) \quad (7)$$

where

$$F\left(a, \frac{R_0}{\rho_0}\right) = a\cdot\frac{R_0}{\rho_0}\int_0^1 e^{-1/2(R_0/\rho_0)^2u^2} J_0(au)\, u\, du. \quad (8)$$

The launching efficiency of the dominant beam mode, i.e., the ratio between the power transmitted in this beam mode and the power transmitted in the metallic waveguide is

$$\eta = C^2\cdot\frac{\int_0^{2\pi}\int_0^{\infty}\left(E_\rho^{(2)}H_\phi^{(2)} + E_\phi^{(2)}H_\rho^{(2)}\right)\rho\, d\rho\, d\phi}{\int_0^{2\pi}\int_0^{R_0}\left(E_\rho^{(1)}H_\phi^{(1)} + E_\phi^{(1)}H_\rho^{(1)}\right)\rho\, d\rho\, d\phi}. \quad (9)$$

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[1] S. I. Baskakow, "Excitation of a beam waveguide," *Radio Engrg. Electronic Phys.*, vol. 9, pp. 492–499, April 1964.

[2] A graph, given in Baskakow's paper, shows an optimum launching efficiency of about 43 percent; apparently this graph is erroneous by a factor 2.

[3] R. J. Chaffin and F. J. Beyer, "A low-loss launcher for the beam waveguide," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-12, p. 555, September 1964.

[4] G. Goubau and F. Schwering, "On the guided propagation of electromagnetic wave beams," *IEEE Trans. on Antennas and Propagation*, vol. AP-9, pp. 248–256, May 1961.

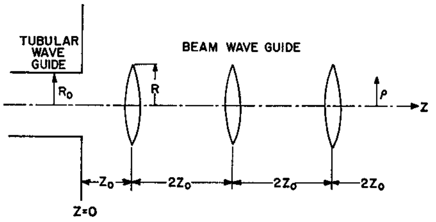

Fig. 1. The launching structure.

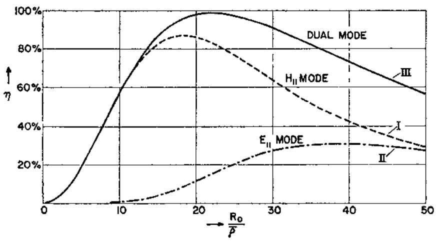

Fig. 2. The launching efficiency η of the dominant beam mode as a function of R_0/ρ_0.

——————— Excitation by a superposition of a H_{11}-mode and an E_{11}-mode (with optimum amplitude ratio of the two modes).
– – – – – – – Excitation by a H_{11}-mode
—·—·—·— Excitation by an E_{11}-mode.

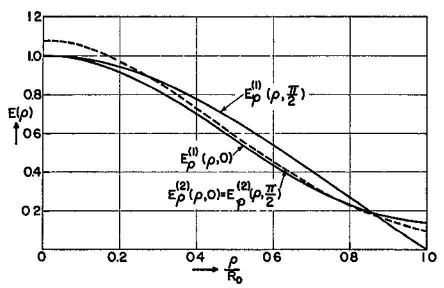

Fig. 3. The cross-sectional electric field distribution in the waveguide aperture in the case of optimum launching conditions for the dominant beam mode. The dotted curves shows the Gaussian field distribution of the beam mode.

Inserting (1) and (4) into (9) the integrals over ρ and ϕ can be evaluated; expressing furthermore C by (7) the power ratio η becomes

$$\eta = \frac{\left\{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right) + \frac{B}{A} F\left(a_{11}, \frac{R_0}{\rho_0}\right)\right\}^2}{\alpha^2 + \left(\frac{B}{A}\right)^2 \cdot \beta^2} \qquad (10)$$

where

$$\alpha^2 = \tfrac{1}{2}(\bar{a}_{11}^2 - 1)\bar{a}_{11}^2 J_0^2(\bar{a}_{11}) \qquad \beta^2 = \tfrac{1}{2} a_{11}^2 J_0^2(a_{11})$$
$$= 0.40458 \qquad\qquad = 1.1908.$$

Equation (10) yields the launching efficiency as a function of R_0/ρ_0 and of the amplitude ratio B/A between the E_{11}- and H_{11}-modes. A simple calculation shows that η as a function of B/A has a maximum for

$$\frac{B}{A} = \frac{\alpha^2}{\beta^2} \cdot \frac{F\left(a_{11}, \frac{R_0}{\rho_0}\right)}{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right)}. \qquad (11)$$

The maximum value itself is with (10)

$$\hat{\eta} = \left[\frac{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right)}{\alpha}\right]^2 + \left[\frac{F\left(a_{11}, \frac{R_0}{\rho_0}\right)}{\beta}\right]^2. \qquad (12)$$

The integrals F (8) on the right-hand side of this equation have been evaluated by a computer as functions of R_0/ρ_0. With the numerical values obtained $\hat{\eta}$ according to (12) has been plotted in Fig. 2. The optimum launching efficiency is reached at $R_0/\rho_0 = 2.15$; the optimum value, as stated before, is 98.3 percent. Note that for a confocal beam waveguide ($z_0 = f$), the optimum aperture radius of the metallic waveguide, according to (5), is $R_0 = 3.04\sqrt{z_0/k}$.

The dotted curves in Fig. 2 show the launching efficiency if the dominant beam mode is excited by only the H_{11}-mode ($B=0$) or by only the E_{11}-mode ($A=0$). In the first case, as stated before, a launching efficiency of 86.7 percent can be achieved; in the second case the launching efficiency does not exceed 31 percent.

In Fig. 3 the tangential electric field distribution in the waveguide aperture is plotted for optimum launching conditions; for $R_0/\rho_0 = 2.15$ the amplitude ratio B/A according to (11) is

$$\frac{B}{A} = 0.250. \qquad (13)$$

(The corresponding power ratio of the E_{11}-mode and the H_{11}-mode is 0.184.) The curves show the components $E_\rho^{(1)}$ and $E_\phi^{(1)}$ as functions of ρ for $\phi = 0$ and $\phi = \pi/2$, respectively; these components have been normalized to unity at $\rho = 0$. For a comparison the Gaussian distribution of the components $E_\rho^{(2)}$, $E_\phi^{(2)}$ of the dominant beam mode in the plane $z=0$ is indicated by the dotted line; their amplitude factor C ($=1.08$) is with (7), determined by the normalization of $E_\rho^{(1)}$, $E_\phi^{(1)}$.

Potter[5] has treated the radiation from the open end of a cylindrical waveguide illuminated by a superposition of a H_{11}-mode and an E_{11}-mode. If the amplitude ratio of the two modes is in the neighborhood of the value (13) a radiation pattern with a low sidelobe level is obtained. The minimum possible backlobe is achieved for an amplitude ratio that differs from the value (13) by less than 2 percent; in this case the sidelobe level is below 44 dB. These results are in agreement with our calculations as the radiation characteristic of a Gaussian wave beam has no sidelobes. Sidelobes must be accounted for by the higher-order beam modes which, in the case of optimum launching efficiency for the dominant beam mode, are excited only with very small amplitudes.

[5] P. D. Potter, "A new horn antenna with suppressed side lobes and equal beam widths," *Microwave J.*, vol. 6, pp. 71–78, June 1963.

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Comments on Loaded Q of a Waveguide Cavity Resonator

Several equations relating the doubly loaded Q of a lossless waveguide cavity resonator to the normalized susceptance of the input/output inductive coupling obstacles (i.e., posts or irises) have appeared in the literature. These equations employ transcendental functions that are not convenient for many design calculations. In this correspondence, an approximate equation will be presented that does not employ transcendental functions and gives reasonably accurate results for most practical cases.

A single cavity resonator symmetrically loaded by matched waveguide terminations through inductive discontinuities can be represented by the equivalent circuit shown in Fig. 1. For a nominal half-wave resonator, resonance occurs when

$$\tan\phi = \frac{2}{b} \qquad (1)$$

where

b = normalized susceptance of discontinuity (Note: b will be *negative* for *inductive* irises or posts)
$\phi = 2\pi l/\lambda_{g0}$ = cavity electrical length
l = cavity physical length
λ_{g0} = guide wavelength at resonance.

Assuming b varies linearly with λ_g (i.e., $B = K\lambda_g$ where K is a constant), the loaded Q of the resonant cavity has been derived by Reed [1]

$$Q_L = \frac{1}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \left[-b\sqrt{b^2+4} \cdot \tan^{-1}\left(\frac{2}{b}\right) + \frac{2b^2}{\sqrt{b^2+4}}\right] \qquad (2)$$

where

Q_L = loaded Q of resonant cavity
λ_0 = free-space wavelength at resonance.

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